Generalized mixers for quantum approximate optimization with non-Hamming weight preserving constraints

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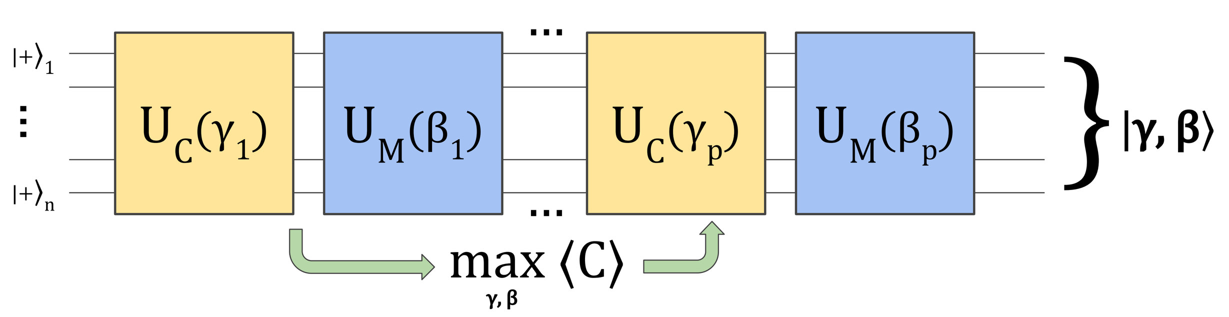
Abstract

The quantum approximate optimization algorithm (QAOA), later generalized to the quantum alternating operator ansatz, has been proposed as a promising technique for approximately solving classically hard problems using quantum computing. The current methods for applying QAOA with constraints have focused on either introducing a penalty term to the cost function or using an "XY"-mixer, which only works for some constraints. In this project, we propose a general technique for procedurally generating QAOA mixers for constrained optimization problems with a class of constraints that generalize the simpler constraints considered in previous works. We determine a small set of rules that allow for transitions between all in-constraint states and translate these into a set of operators. We use the sum of these state transition operators as the mixing Hamiltonian for QAOA, which restricts the solutions to within the constraint space.

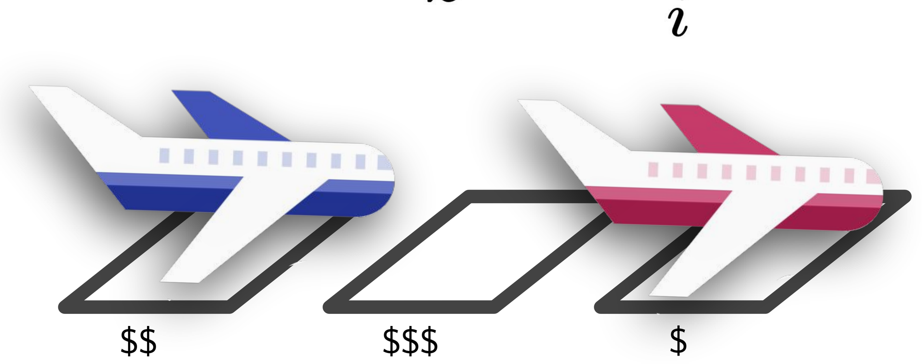
Overview of QAOA



QAOA relies on layers of parameterized Hamiltonian evolution under a cost Hamiltonian that marks good solutions and a mixer Hamiltonian that drives transitions between solutions, with classically-computed parameters. For a given optimization problem defined by a cost function, QAOA converges to the optimal solution.

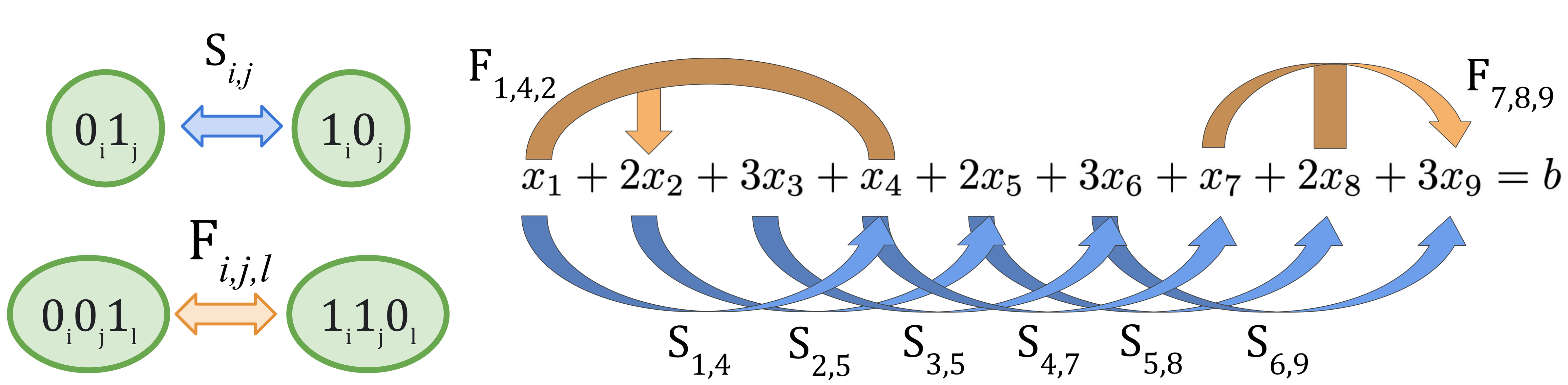
Ex: minimizing cost of docking airplanes.
Constraint: one airplane per bay

$$z_{\text{opt}} = \arg\min_{z} \sum_i c_i z_i$$



Constraint Preserving Mixers for Binary Integer Linear Programming (BILP)

We develop a set of state transition operators similar to those shown below to solve BILP problems with a single constraint equation with N variables, whose coefficients are integers from 1 to k, repeating.

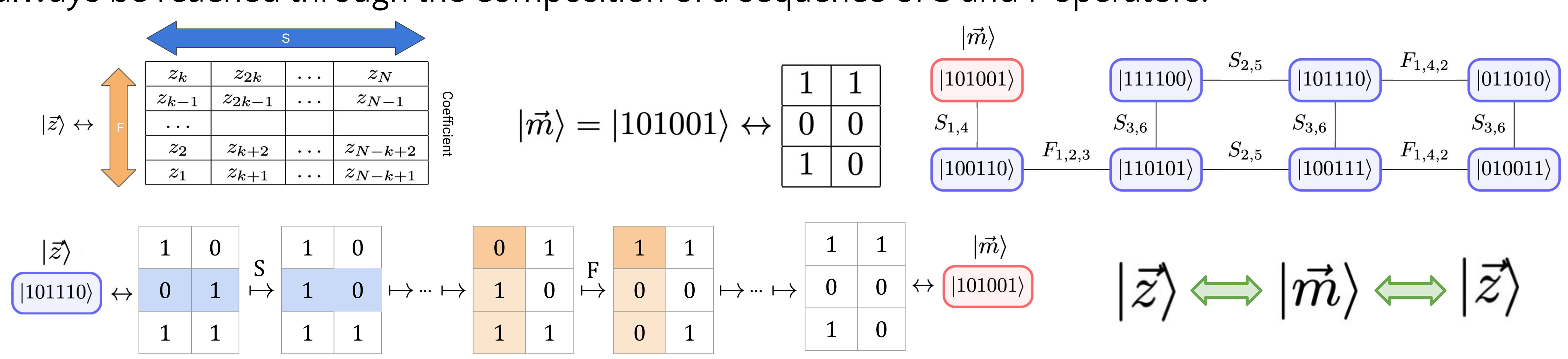


Conclusions

- We developed a technique for developing operators that enforce constraints for QAOA
- We prove connectivity properties essential for convergence
- Simulations show good performance accross a variety of different sizes

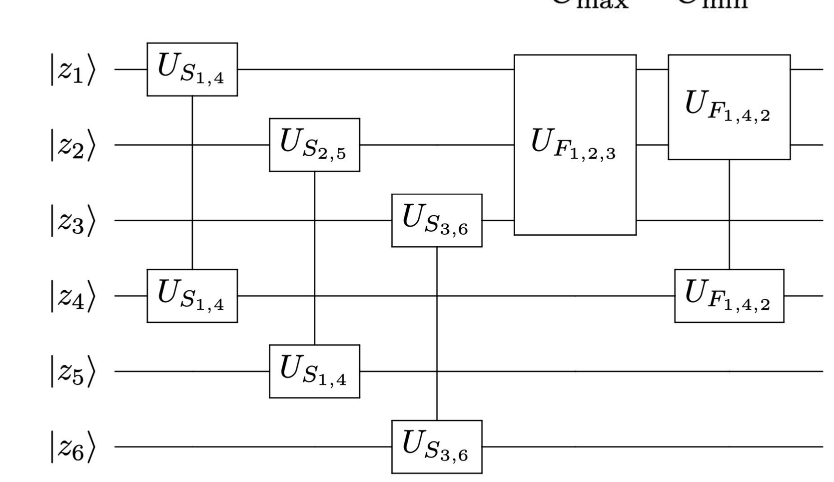
Proof of Connectivity

An important step in proving the convergence of the algorithm to the optimal solution is showing that all in-constraint states can be accessed using the transition operators. We developed an algorithm to systematically move from one state to another, as visualized in tables below. We show that for any BILP problem of this form, the state, $|\vec{m}\rangle$,which has ones as far up and to the left in the table as possible, can always be reached through the composition of a sequence of S and F operators.

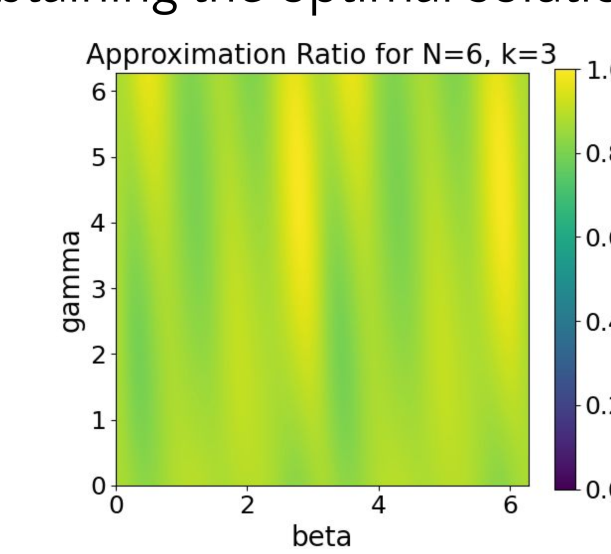


Experimental Results

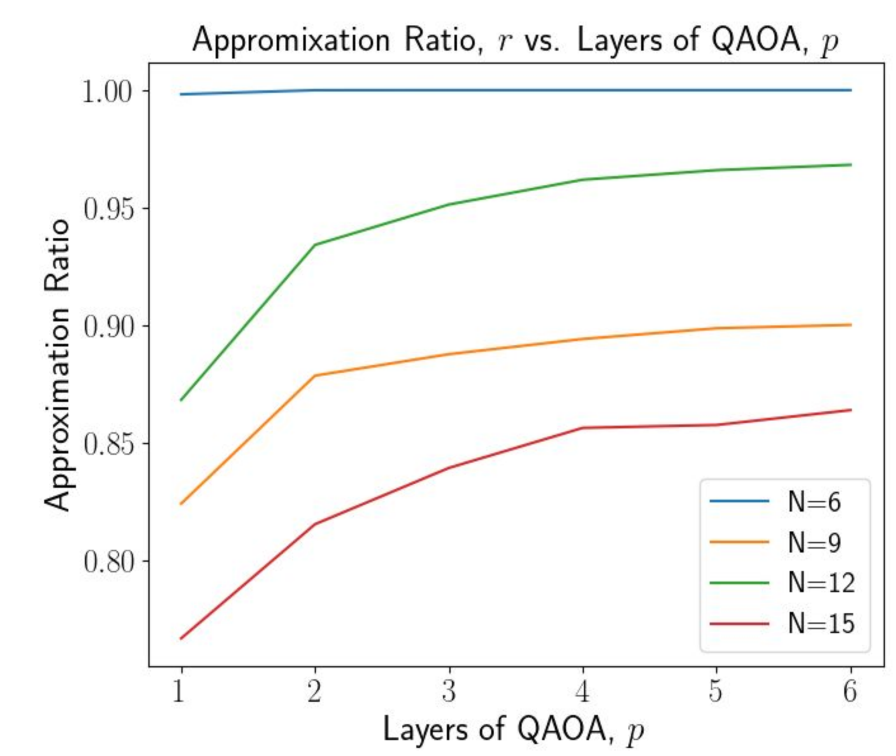
We evaluate the empirical performance of the mixer via classical simulations of QAOA. We use metrics such as approximation ratio, $r = \frac{\langle C \rangle - C_{\min}}{C_{\max} - C_{\min}}$, and the probability of obtaining the optimal solution, z_{opt}.

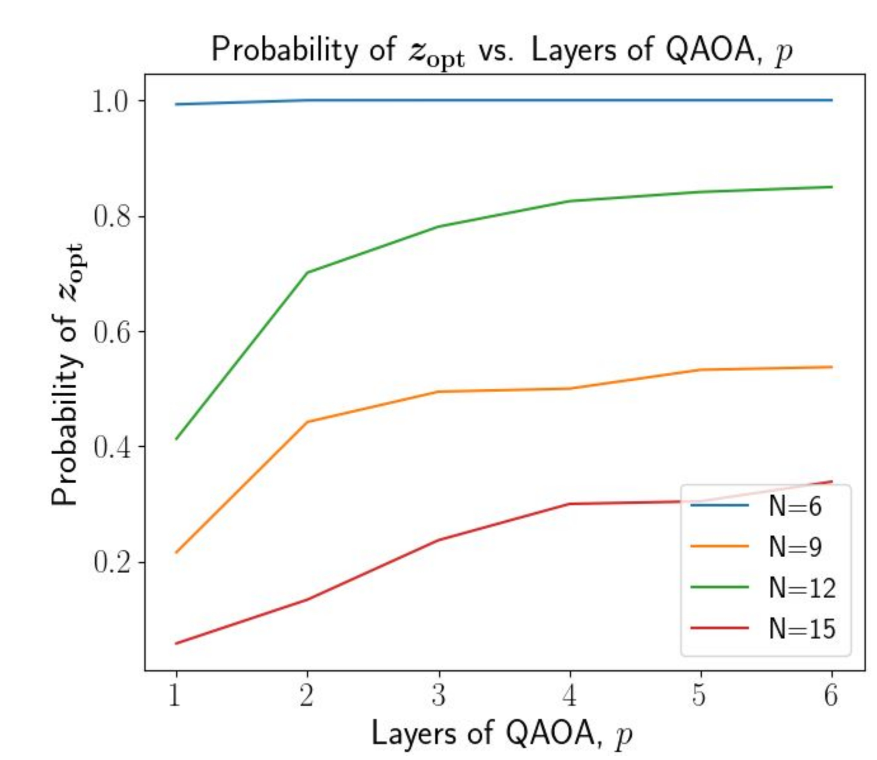


Quantum circuit diagram of a mixer within a single layer of QAOA for the for the constraint N=6, k=3:
$x_1 + 2x_2 + 3x_3 + x_4 + 2x_5 + 3x_6 = b$



Approximation Ratio landscape for 1-layer QAOA





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